[wider]article document **The statistics transmuting Chern-Simons field and the braid group on Riemann surfaces of genus** $g > 0$ Ansar Fayyazuddin

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Abstract

We study bosons interacting with an abelian Chern-Simons field on Riemann surfaces of genus $g > 0$. It is shown that a singular gauge transformation brings the hamiltonian to free form. The transformed wave functions furnish a multi-component representation of the braid group studied by Imbo and March-Russell. The construction constitutes a proof of the equivalence of bosons coupled to a Chern-Simons field and anyons and generalizes the well known equivalence of the two pictures on the plane.

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Introduction In 2+1 dimensions particles can have statistics other than Bose or Fermi. Particles which obey such statistics are called anyonslm,wil. Since the wave function picks up a complex phase when two particles are exchanged it is not enough to say that "two particles are exchanged" but one must also specify how they are exchanged (clockwise or anti-clockwise, along a path enclosing or not enclosing other particles etc.) This amounts to saying that the wave function must provide a representation of the braid group. The braid group is essentially the group of particle exchanges which keeps track of the path of exchange.

It has been known for some time that on the plane one can study the problem of anyons by coupling bosons to a Chern-Simons field. The equivalence of the two pictures is established by constructing a transformation which relates one picture to the other. This equivalence has only been proved for the plane where the global topology of the space is trivial and for the torusans. It is the purpose of this paper to establish this equivalence for space-times of the form $\Sigma_g \times R$, where Σ_g is a a compact Riemann surface of genus $g > 0$. We know from studies of the braid group on Riemann surfaces of arbitrary genus that the translation properties of anyon wave functions become quite complicated as one increases the genus. In particular, in addition to the transformations which exchange particles around homologically trivial paths one also has to take in to account translations along the handles of the surface Σ_g. Interestingly, translations around the two homology cycles of a handle do not in general commute. So wave functions can not always provide a single-component representation of the braid groupeinar,imr,wen,lee.

The paper is structured as follows. In sect. 2 we collect a few relevant results from the theory of Riemann surfaces. In sect. 3 the Chern-Simons constraint is solved and the remaining degrees of freedom are quantized. The first quantized hamiltonian is derived in sect. 4 and the dependence of the wave function on the topological component of the gauge field is completely determined. Finally, in sect. 5 we show that the hamiltonian can be brought to free form by a singular gauge transformation. The gauge transformed wave functions furnish a representation of the braid group studied by Imbo and March-Russell imr.

Mathematical Background To make the discussion self-contained we state a few mathematical results here which will be needed in the rest of the paper. Most of the results stated here can be found in standard mathematical references on Riemann surfaces riem and in the physics literature vv,gaume where concise and readable accounts of the main results are presented. We will work on a compact Riemann surface

of genus g, which we will denote by Σ_g with $g > 0$. We equip Σ_g with the canonical homology basis $\{a_i, b_i\}$ shown in Fig. 1. Associated with this homology basis is a basis of holomorphic abelian differentials $\omega_i = \mu_i(z)\, dz$. They are completely determined by the requirement equation $\int_{a_i} \omega_j = \delta_{ij}$.